Chapter 3:
Synchronous Generator Modeling

**Internal Generated Voltage:**

The rotor has an electromagnet controlled by a field current, always pointing in the direction of the magnet. When the rotor rotates, an induced voltage appears in the stator windings. Equation 2-3 can be rewritten as:

\[ E = k \Phi \omega \]  

[3-1]

Where \( \Phi \) is the main flux provided by the excitation, and \( \omega \) is the angular speed of the rotor and \( k \) is a constant dependent upon the machine geometry.

Note that the electrical frequency produced \( f_e \) is related to the number of poles \( P \) and the speed of the rotor \( N \) by:

\[ f_e = \frac{N_{rpm} \text{Poles}}{120} \]  

[3-2]

Practically the NO-LOAD voltage at rated speed for an increasing excitation current is recorded as shown by figure 3-1.

As seen, this is similar to the saturation curve of a magnetic material, which it is!
Note also that this curve is drawn at RATED SPEED. According to equation 3-1, this no-load characteristic is proportional to the speed of the rotor.
Cylindrical Model (Behn Eschenburg)

The main assumption is that the rotor is smooth that means that the reluctance of the air gap is constant and does not depend upon the position of the rotor.

The rotor produces a flux $\Phi_f$, and when the shaft rotates at synchronous speed one can draw the equivalent circuit on a **PER PHASE BASIS** (assumes that the 3 phases are balanced). $R_a$ is called the “effective resistance” of the armature winding. This is about 1.3 times the DC resistance because it takes into account the AC resistance due to skin effect caused by the AC current at 50Hz. $X_a$ is the “leakage reactance” of the armature winding, caused by the flux linking the windings. This flux does not link with the field winding, hence does not produce any voltage. (This is a combination of several effects: end connection; leakage reactance; slot leakage reactance; tooth top and zigzag (or differential) leakage reactance; belt leakage reactance). These are advanced topics that we are not covering here.

The synchronous generator can be shown as

![Figure 3-2](Image)

One can derive the equation

$$E_f = V_a + I_a R_a + jI_a X_a$$

[3-3]

Corresponding to the phasor diagram

![Figure 3-3](Image)
Note the “power factor” angle $\theta$ which in this case is LAGGING (the current lags the voltage = inductive load). Note also the angle between $V$ and $E$ ($\delta$) called the “internal angle” which will play an important role.

Note also the cases where the power factor is unity and leads

![Figure 3-4](image)

In most cases the resistance of the armature is negligible w.r.t. the leakage inductance, and in a first approximation one can draw the phasors without it. If we maintain the excitation constant, and only the current varies, we have the following cases:

![Figure 3-4](image)

Shows that for a power factor unity and an increasing current, the output voltage decreases

For a lagging power factor this effect is much more pronounced
And for a leading power factor, the output voltage INCREASES with the current and is larger than the internal voltage.

Figure 3-5

ARMATURE REACTION

The flux produced by the armature winding reacts with the flux set up by the poles on the rotor. The total flux will therefore be reduced. This is called the **armature reaction**. With reference to figure 3-6 let us examine a sequence of events when the generator delivers a load at unity power factor:

Figure 3-6

a) If $\Phi_p$ is the flux under a pole at no load, the internal generated voltage $E_a$ must lag by 90°.

b) At unity pf, the phase current $I_a$ is in phase with the terminal voltage $V$.

c) As phase current $I_a$ passes through the armature winding, its magneto motive force (mmf) produces a flux $\Phi_{ar}$ which is in phase with $I_a$. The effective flux $\Phi_e$ per pole in the generator is therefore

$$\Phi_e = \Phi_p + \Phi_{ar} \quad [3-4]$$
d) The flux $\Phi_{ar}$, in turn induces an emf $E_{ar}$ in the armature winding. It is called the \textbf{armature reaction emf}. It lags the flux $\Phi_{ar}$ by 90° hence the effective voltage $E_e$ per phase is:

$$E_e = E_a + E_{ar} \quad \text{[3-5]}$$

e) The equivalent circuit can be shown and the equation derived as:

$$E_e = V_a + I_a(R_a + jX_a) \quad \text{[3-5]}$$

\textbf{NOTE:} both magnetizing and leakage reactances are present at the same time, but it is rather difficult to separate one from the other. It is simpler to combine them

$$X_s = X_m + X_a \quad \text{[3-6]}$$

and call it the \textbf{SYNCHRONOUS REACTANCE}

We can also define the \textbf{SYNCHRONOUS IMPEDANCE} $Z_s = R_a + jX_s \quad \text{[3-7]}$

\textbf{Note:} $R_a$ can be measured with DC measurement techniques, and has to be corrected to AC value which is approximately a factor of 1.3. However, this AC value is still much smaller than the value of the synchronous reactance of the machine.

For illustration purposes, let us see how the diagram evolves with the various power factor situations.
A synchronous generator is driven by a mechanical prime mover, and has to have a constant speed, or else the frequency delivered will wander. In North America in most places, the frequency has to be maintained within 0.05 Hz.

The total output power generated is expressed as:

\[ P = 3V_{\text{phase-neutral}} I_{\text{phase}} \cos \theta \]  \hspace{1cm} [3-8]  \hspace{1cm} \text{(phase quantities)}

or

\[ P = \sqrt{3}V_{\text{line-line}} I_{\text{phase}} \cos \theta \]  \hspace{1cm} [3-9]  \hspace{1cm} \text{(line-to-line)}
Similarly the reactive power output is:

\[ Q = \sqrt{3}V_{\text{line-line}}I_{\text{phase}} \sin \theta \]  \[3-10\]

As mentioned previously, most of the time the term RI can be neglected w.r.t. the jXI term, and this brings a further simplification which is very useful. In figure 3-10 one can see that the projection of jXI on the V axis is equal to the projection of E on the axis perpendicular with V.

\[ X \cos \delta = E \sin \delta \]

Figure 3-10

Hence:

\[ P = 3VI \cos \theta = \frac{3VE \sin \delta}{X} \]  \[3-11\]

The equation above shows clearly that a given generator cannot convert more power than:

\[ P_{\text{max}} = \frac{3VE}{X} \]  \[3-12\]

This is called “the static stability limit” of the generator.

Note in the figure below where the active power and reactive power are found (appropriate scaling for \( k=3V/X \))
MEASUREMENT OF THE PARAMETERS

The parameters that are needed to model the machine with the cylindrical model are:

- The E(i) curve (no-load)
- The synchronous reactance X
- The armature resistance R

The armature resistance is measured as a DC resistance with a voltage/current reading. The measured value has then to be adjusted for AC value, taking account of skin effect. It is usually 1.2 to 1.5 times higher than the DC resistance.

The generator is then driven at rated speed and the open circuit voltage (or no-load) is recorded while increasing the excitation current of the generator. The curve E(i) is obtained.

The generator, still running at rated speed is then short circuited, and the current is measured while slowly increasing the excitation. By examining the equivalent circuit of figure 3-2, one can see that

\[ E = Rl + jXl \]  

Hence the short circuit impedance

\[ Z = |E| \]  

for an excitation of (i)

This points out clearly that X is non linear and depends upon the excitation and load operating points. For practical applications, since this is anyway a simple model, the value of X is approximated by first neglecting the resistor (at least 50 times smaller than X), and confining the studies in 2 ranges: the “unsaturated range” and the “saturated range”. Hence from Figure 3-11 where the abscissa is the excitation current and the ordinate has one scale: no load voltage (internal voltage or EMF), called the “open circuit characteristic), and the other has the scale of the armature current, called the “short circuit characteristic”.
The unsaturated value can be calculated from the air gap line on the figure as:

$$X_u = \frac{ad}{cd} \quad [3-15]$$

And the saturated SYNCHRONOUS REACTANCE is

$$X_s = \frac{bd}{cd} \quad [3-16]$$

Usually one takes the measurement of the open circuit voltage at rated excitation current and the short circuit current at that rated excitation current

$$X_s = \frac{V_{oc}}{I_{sc}} \quad [3-17]$$

One can also obtain the synchronous reactance when the machine saturates. The method is called method of POTIER, and provides the “Potier Synchronous Reactance” to be used in saturated conditions.

To the tests performed on figure 3-12, we add a test called “Wattless”. The generator feeds a purely inductive load, and we need one point (W) which gives rated voltage at an excitation $i_{scw}$, and we record the corresponding current $I_w$. We then make a short circuit test recording the excitation current $i_{scw}$ that produces a short circuit current $I_w$. 

Figure 3-12
Figure 3-13
METHOD:

- Record the open circuit characteristic.
- Measure the excitation current $i_{ecw}$ which gives rated voltage on a purely inductive load (wattless) >>> W on the diagram, and record the current $I_w$
- Measure the excitation current $i_{scw}$ that gives a short circuit current of $I_w$

Point A is obtained by reducing the excitation value at W by the excitation $i_{scw}$ that provides a short circuit current identical to that of the wattless test.
- Draw the line parallel to the air gap line passing through A >> point B

- The **triangle ABW** is called the “**Potier triangle**”, and its height, BH represents the “Potier voltage” \( U_p = X_p I_a \)

Hence the corrected synchronous reactance is: \( X_{sp} = X_s + X_p \)

The value obtained with this method is higher than the saturated reactance obtained from equation 3-17, and leads to slightly higher voltage drop for the saturated machine.

In the next figure 3-13, we can see the voltage regulation of the generator expressed as in equation [] depending upon the power factor of the load

![Figure 3-13](image)
Salient Pole Synchronous Generators

The cylindrical rotor seen so far showed a uniform air gap. However, in certain machines, poles are built with localized poles (figure 3-14 shows a 2 pole rotor). This means that the reluctance path of the air gap varies according to the position of the rotor.

On figure 3-15 we can see that the flux density is maximum on the axis that passes through the center of the pole. This axis is called the **DIRECT AXIS** and this axis rotates with the rotor. However one can see that at the flux lines are thinning as we go towards the edge of the pole, the magnetization decreases, and the flux density is smaller.

We can now define an axis, perpendicular to the direct axis on the rotor surface, called the **QUADRATURE AXIS**. Since the reluctance of a circuit is defined as the magneto motive force over the flux \( \mathcal{R} = \frac{\mathcal{J}}{\Phi} \), we can see that this reluctance varies with the position of the rotor. We can now make the assumption that we can model the machine in both axis reference frame, and **superpose the 2 models**.

One can now define a synchronous reactance on each of the axes, namely \( X_d \) and \( X_q \). Similarly the armature current can be resolved in the 2 models. Let us now study the effect of the armature reaction in each model:

**Direct axis model: Figure 3-16**

Note that the internal voltage \( E \) lags the field flux by 90 degrees \( \left( e = -\frac{d\Phi}{dt} \right) \).
The armature current is in phase with the flux \( \Phi = kI \), hence the direct axis model projects the current on the flux direct axis as \( I_d \), and one can see the armature reaction \( \Phi_{ard} \) due to the direct axis current. Again the voltage resulting from this flux reaction \( \Phi_{ard} \) lags by 90° therefore \( E_d \) is shown to decrease the direct no-load voltage \( E \).

![Diagram of Synchronous Generator Modeling](image)

**Figure 3-16**

Quadrature axis model:

Similarly one projects the current on the quadrature current axis (flux) as \( I_q \) which results in the quadrature armature reaction \( \Phi_{arq} \), rotated by 90° gives the voltage reduction \( E_q \).

![Diagram of Synchronous Generator Modeling](image)

**Figure 3-17**

*Synchronous Generator Modeling* 14
Finally we superpose the 2 models to obtain the effective internal voltage $E_{ef}$

![Figure 3-18]

Practically, instead of using the flux analysis, we can revert to standard circuit model, with voltage drops as (XI). Note however that $X_d$ is always larger than $X_q$, and we also have to take into account the resistive drop

![Figure 3-19]

**POWER OUTPUT**

Each model will have a power output as the product of the terminal voltage $V$ by the projection of the respective current on the voltage. Referring to figure 3-19 and assuming $R=0$,

\[ P_d = 3VI_d \cos(90 - \delta) \quad \text{and} \quad P_q = 3VI_q \cos\delta \]
Since \( I_d = \frac{E - V \cos \delta}{X_d} \) and \( I_q = \frac{V \sin \delta}{X_q} \)

\[
P = 3V \frac{E - V \cos \delta}{X_d} \sin \delta + 3V \frac{V \sin \delta}{X_q} \cos \delta
\]

simplifies once we consider \( \sin y \cos y = 0.5 \sin 2y \)

\[
P = \frac{3VE}{X_d} \sin \delta + \frac{3V^2}{2} \left( \frac{X_d - X_q}{X_d X_q} \right) \sin 2\delta \quad [3-18]
\]

This is a very important equation, since the first term equates to the power of the cylindrical model, and the second term is an extra power that is NOT dependent upon the excitation of the machine. It is called the **reluctance power**, and it depends upon the “saliency”, the difference between the reactances of the direct and quadrature axis. This extra power enables salient pole generators to convert more power as seen by the torque comparisons, and show that salient pole machines have a higher static stability than cylindrical machines.

![Figure 3-20](image-url)